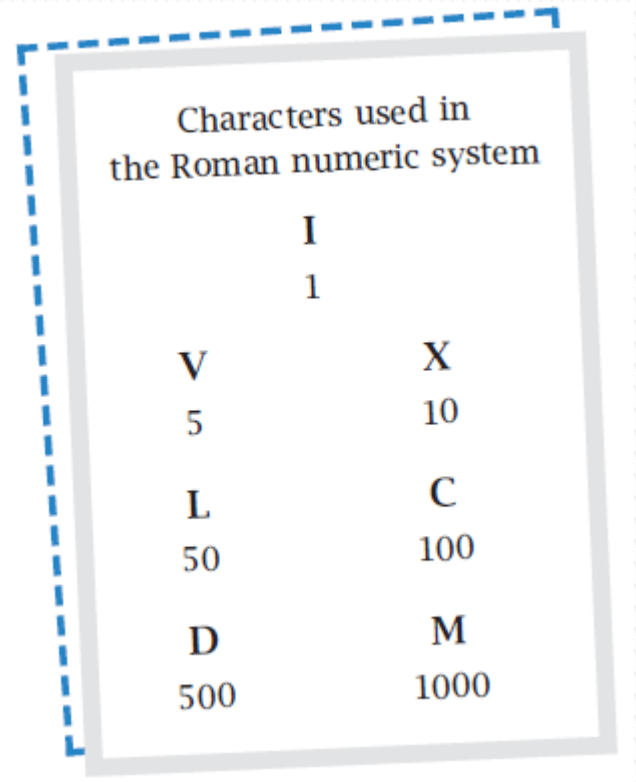
MATHEMATICS

for elementary school

Numbers

- Numbers 0, 1, 2, 3, 4, *. . . are called natural numbers.*
- Numbers *. . . , −3, −2, −1, 0, 1, 2, 3, . . . are called integers.*
- Numbers that can be represented as a fraction *l:m*, where *l and m are integers and* $m \neq 0$*, are called* ***rational numbers.***.

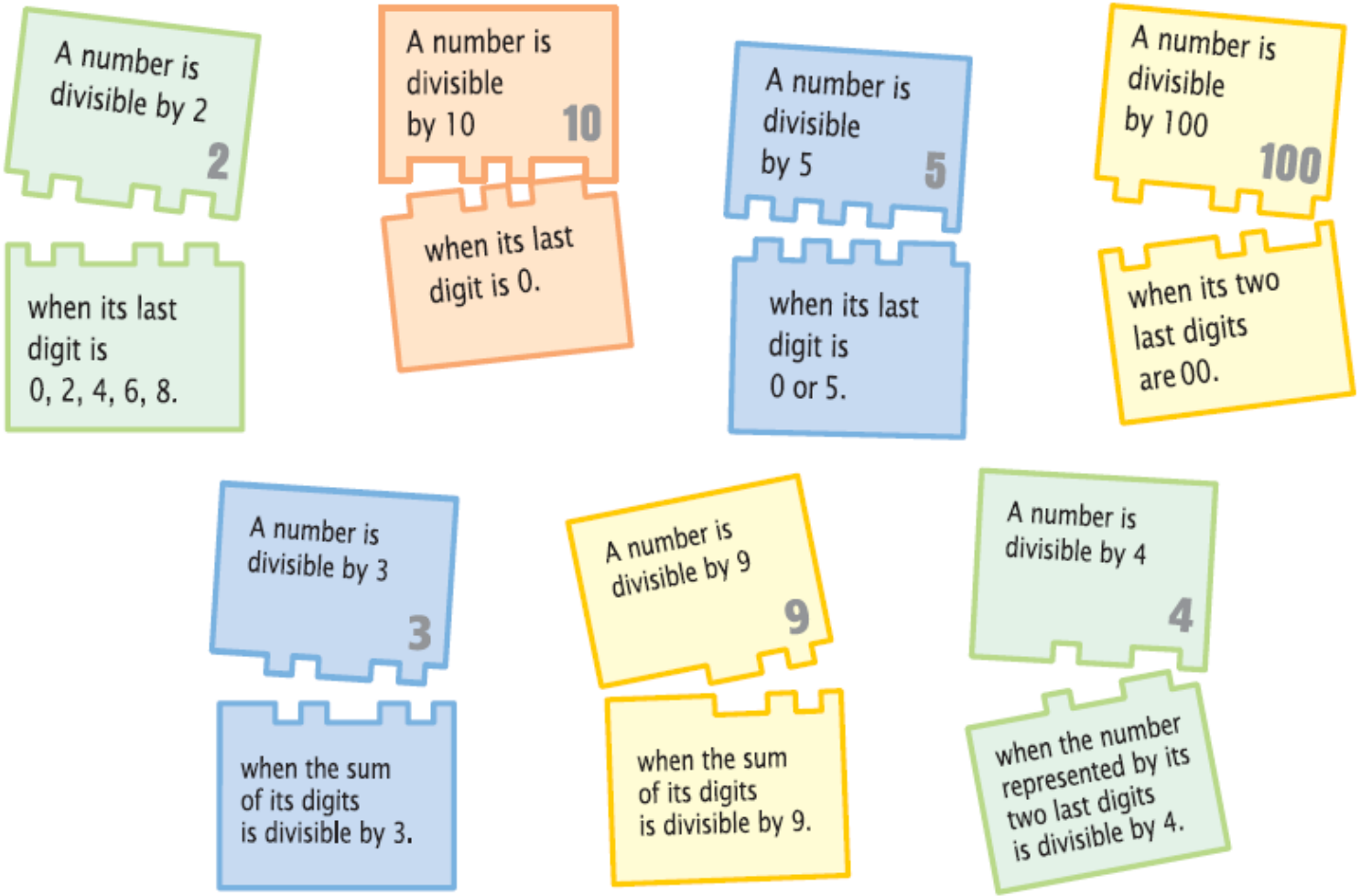
Roman numerals



The rules that apply for that are:

- A maximum number of repetitions of digit I (1), digit X (10), digit C (100) and digit M (1000) standing next to each other is three; other digits can occur only once.
- If there is a smaller digit preceding bigger digit, the value of both is equal to the difference of them; this can occur only in six instances: IV (4) IX (9) XL (40) XC(90) CD(400) CM(900)

The divisibility rules



Percentages

- The word *percent comes* from the Latin expression *pro centum and means "per hundred".*
- One percent of the given quantity is one hundredth of that quantity. Percentages are fractions with denominator 100 written in a different way.

$$1\% = \frac{1}{100} = 0{,}01 \qquad 13\% = \frac{13}{100} = 0{,}13$$

$$6\% = \frac{6}{100} = 0{,}06 \qquad 130\% = \frac{130}{100} = 1{,}3$$

6% of men
means
$\frac{6}{100}$ of all men.

Generally:
p% of the given quantity
is the same as
$\frac{p}{100}$ of that quantity.

$$p\% = \frac{p}{100}$$

Sometimes in various contexts you can meet the symbol ‰ (read permil). The word permil comes from the Latin *pro mille and means "per* thousand".

- One permil of a given quantity is its thousandth part.

$$1‰ = \frac{1}{1000} \qquad 7‰ = \frac{7}{1000} \qquad 2{,}5‰ = \frac{2{,}5}{1000}$$

Because $1‰$ is $\frac{1}{1000}$, and 1% is $\frac{1}{100}$, so $1‰$ is 10 times less than 1%, that is:

$$1‰ = \frac{1}{10}\% \qquad 1\% = 10‰$$

5‰ women is $\frac{5}{1000}$ of all women.

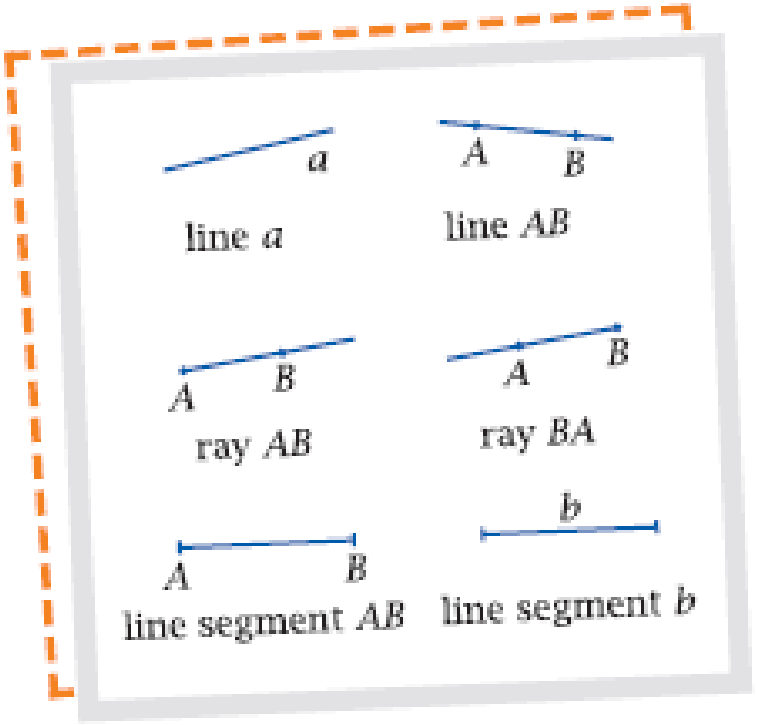
Generally:
$k‰$ of a quantity is $\frac{k}{1000}$ of that quantity.

$$k‰ = \frac{k}{1000}$$

GEOMETRIC FIGURES

- Lines and line segments
- Angles
- Triangles
- Quadrilaterals
- Regular polygons

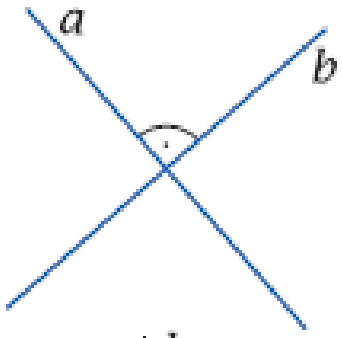
Lines and line segments



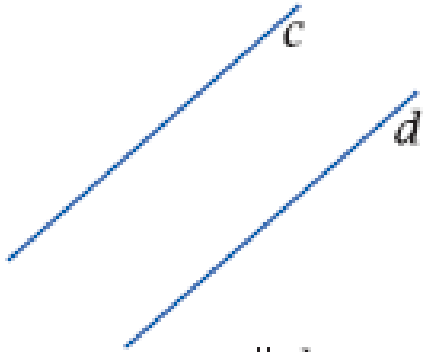
Lines and line segments

Two lines lying on the plane can intersect or have no common points.

- About two lines that intersect at right angles we say that they are perpendicular.
- Of two lines that have no common points we say that they are parallel.



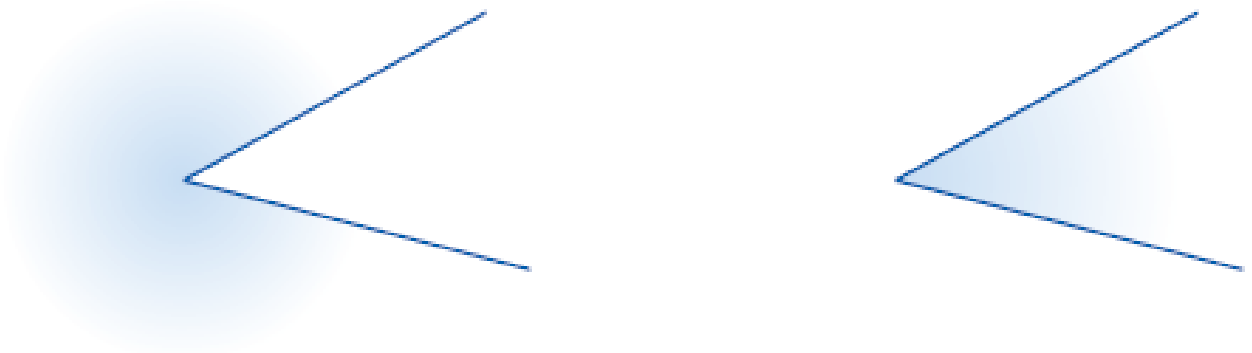
$a \perp b$



$c \parallel d$

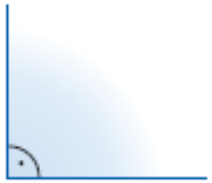
Angles

- Two rays with a common endpoint dissect the plane into two parts.Each of these parts together with the rays is a geometric figure called angle.



The rays forming the angle are called the angle's arms, and their common point — the angle's vertex.

Types of angles



Right angle
Measure 90°.

Straight angle
Measure 180°.

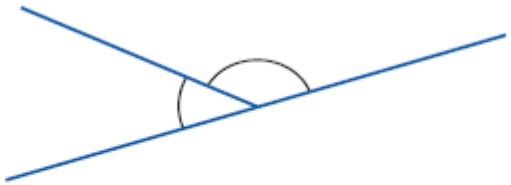
Full angle
Measure 360°.

Acute angles
Measure less than 90°.

Obtuse angles
Measure between
90° and 180°.

Reflex angles
Measure between
180° and 360°.

Types of angles

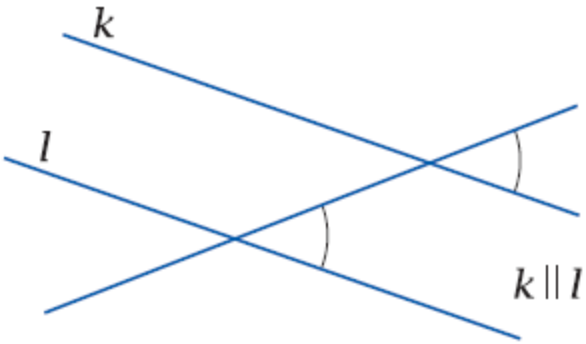


Supplementary adjacent angles

The sum of their measures is 180°.

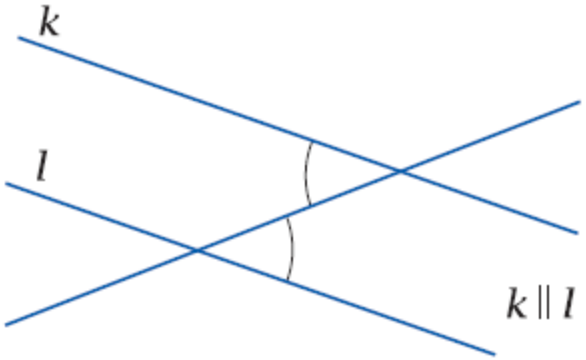
Apex angles

Equal measures



Corresponding angles

Equal measures.

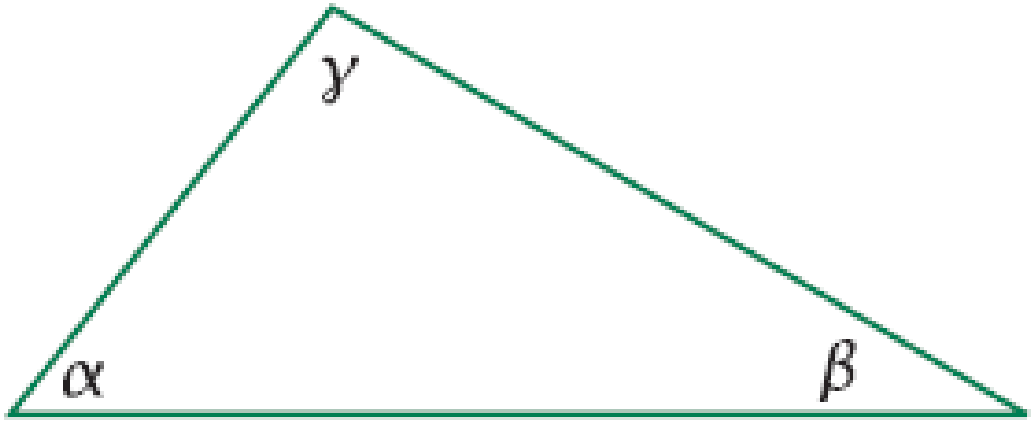


Alternating angles

Equal measures.

Triangles

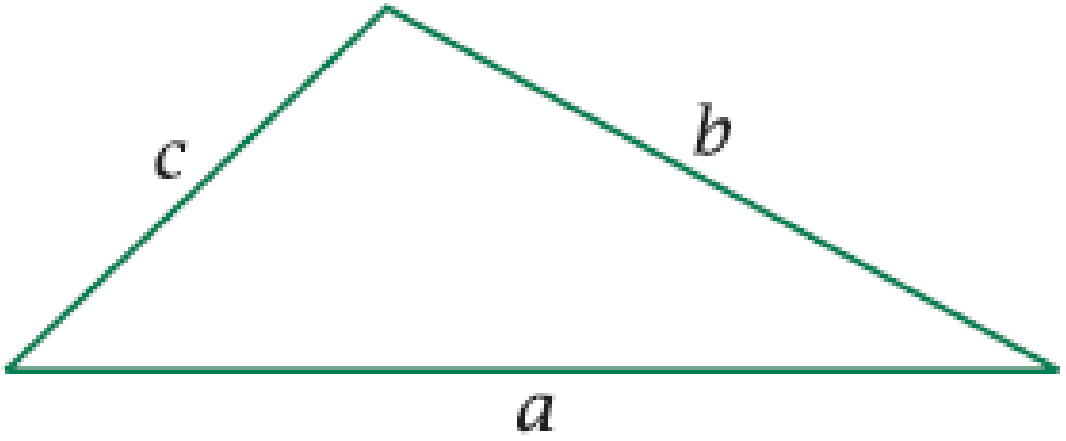
- The sum of the angle measures in a triangle is 180°.



$$\alpha + \beta + \gamma = 180°$$

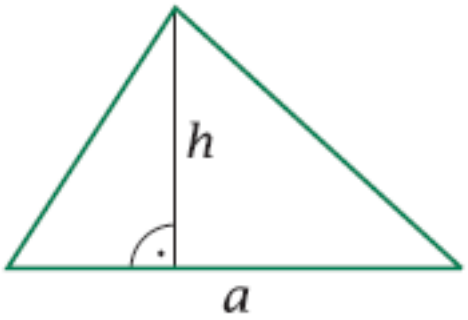
Triangles

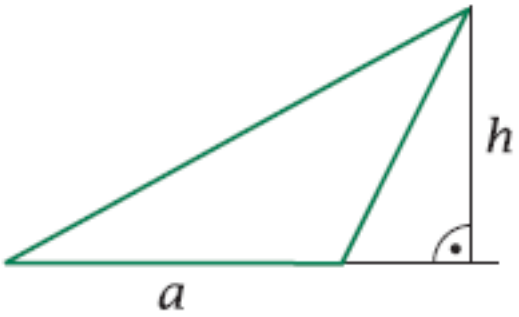
- Each side of a triangle has a length less than the sum of the lengths of the other two sides.

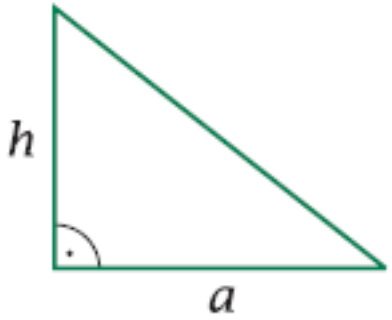


$$a < b + c \qquad b < a + c \qquad c < a + b$$

Formula for the area of a triangle



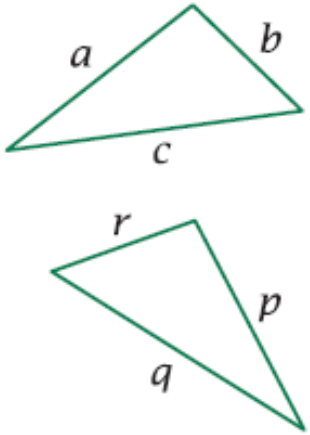
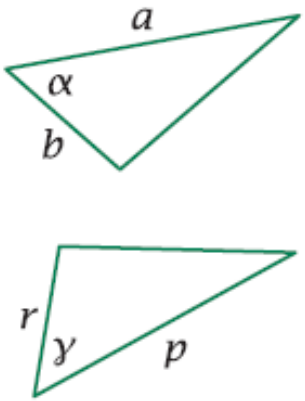
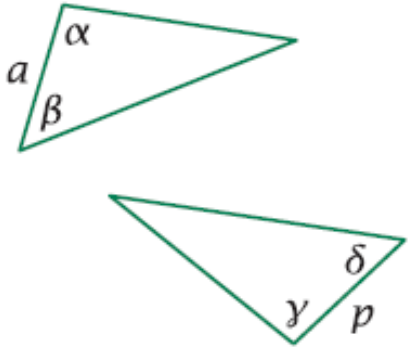




$$A = \frac{a \times h}{2}$$

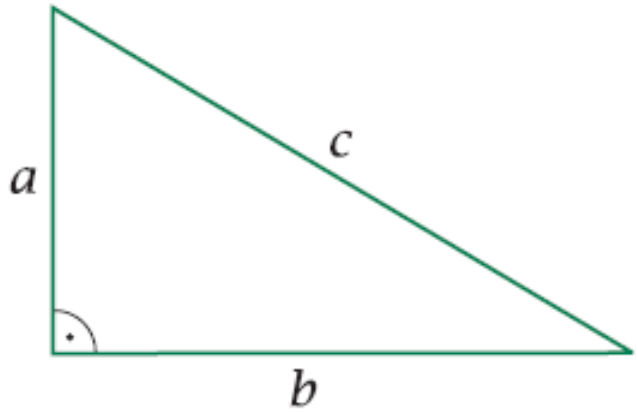
Theorems on congruent triangles

- If the two triangles meet the conditions written under one from the drawings below, the triangles are congruent.

SSS condition (side-side-side)	SAS condition (side-angle-side)	ASA condition (angle-side-angle)
		
$a = p$ $b = r$ $c = q$	$a = p$ $\alpha = \gamma$ $b = r$	$\alpha = \gamma$ $a = p$ $\beta = \delta$

The Pythagorean theorem

- ***If the triangle is right-angled, the sum of the square lengths of the catheti is equal to the square of the hypotenuse's length.***



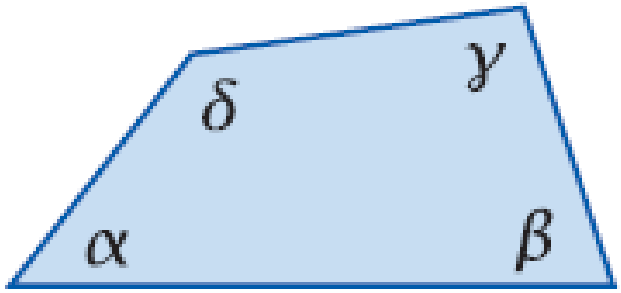
$$a^2 + b^2 = c^2$$

a, b — the lengths of the catheti

c — the length of the hypotenuse

Quadrilaterals

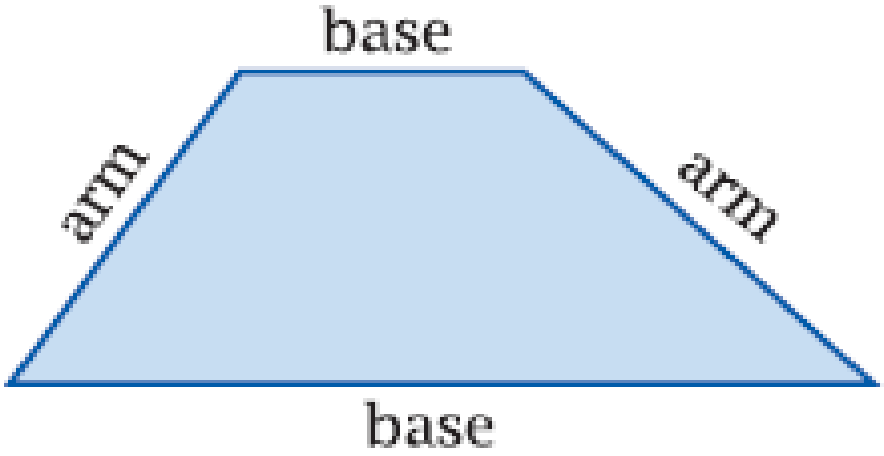
- *The sum of all the four angles of a quadrilateral is 360°.*



$$\alpha + \beta + \gamma + \delta = 360^\circ$$

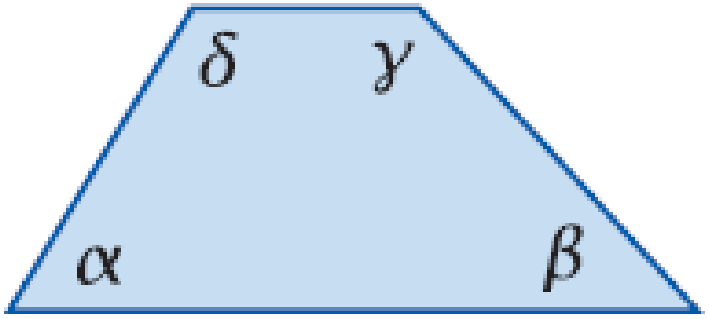
Quadrilaterals – trapezoid

- Trapezoid is a quadrilateral with at least one pair of parallel sides.
- The parallel sides of the trapezoid are called bases and the other sides — arms.
- A trapezoid in which the arms are of equal length is called an isosceles trapezoid.
- A trapezoid that has at least one right angle is called a right-angled trapezoid.



Trapezoid

- *The sum of the angles adjacent to the same arm of a trapezoid is equal to 180°.*

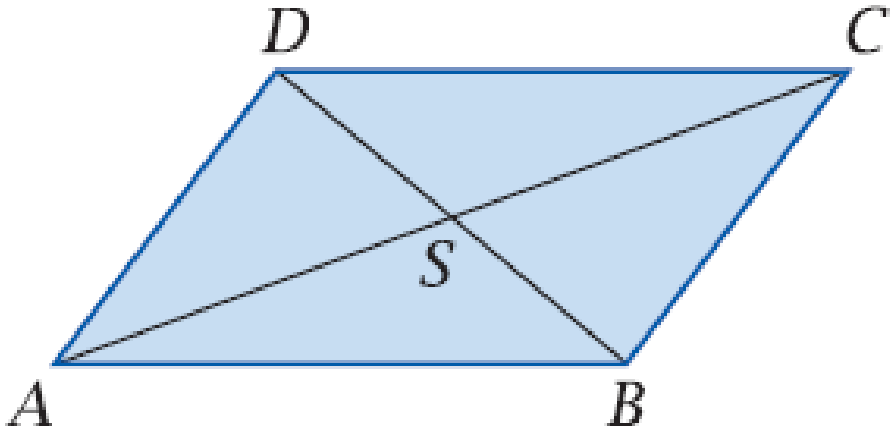


$$\alpha + \delta = 180^\circ$$

$$\beta + \gamma = 180^\circ$$

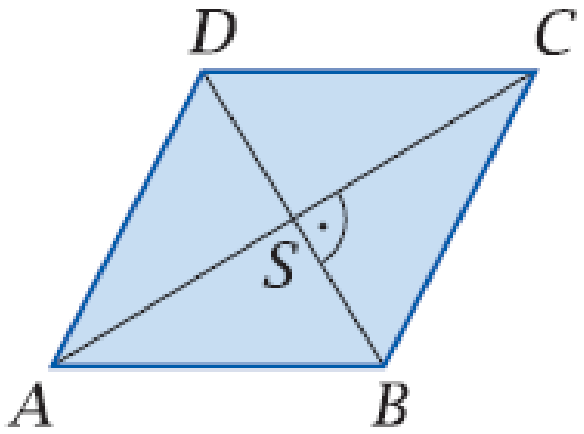
Quadrilaterals – parallelogram

- Parallelogram is a quadrilateral, which has two pairs of parallel sides.
- *The diagonals of a parallelogram intersect at their midpoint.*



Quadrilaterals – rhombus

- Rhombus is a quadrilateral that has all sides of equal length.
- Each rhombus is a parallelogram.
- *Diagonals of a rhombus intersect at their midpoint and are perpendicular.*



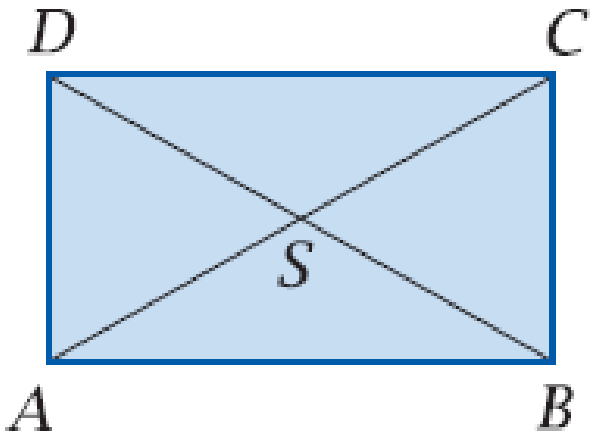
$AC \perp BD$

$AS = SC$

$BS = SD$

Quadrilaterals – rectangle

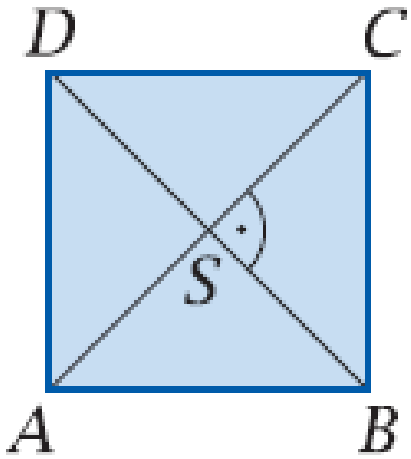
- Rectangle is a quadrilateral whose all the angles are right angles.
- *Diagonals of a rectangle are of equal length and intersect at the midpoint.*



$$AC = BD$$
$$AS = SC$$
$$BS = SD$$

Quadrilaterals – square

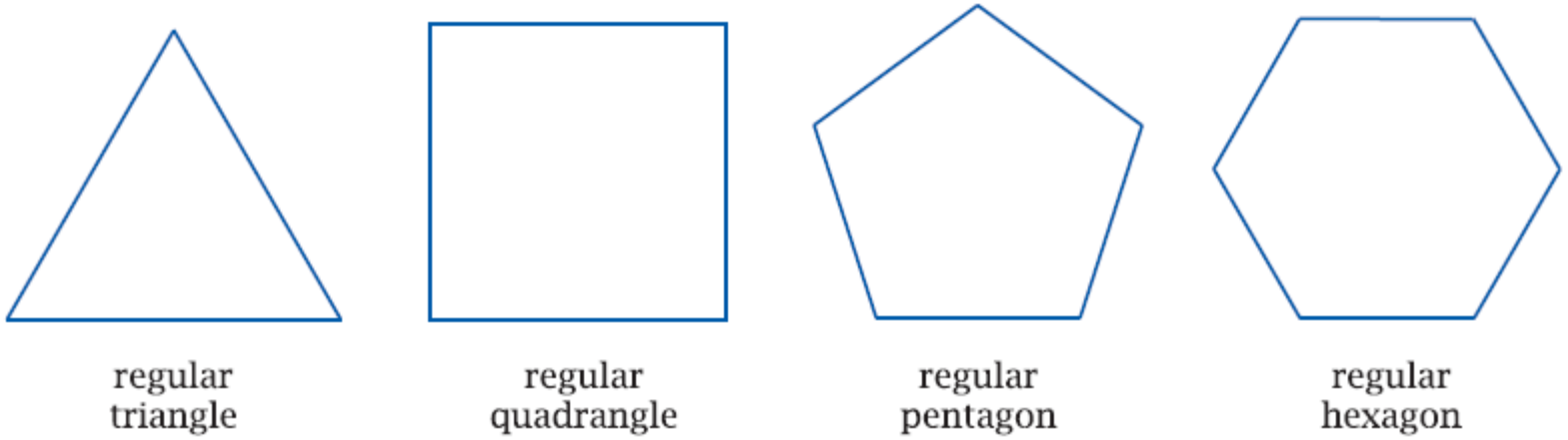
- Square is a rectangle that has all sides of equal length.
- *Diagonals of a square are of equal length, intersect at the midpoint and are perpendicular.*



$$AC \perp BD$$
$$AC = BD$$
$$AS = SC$$
$$BS = SD$$

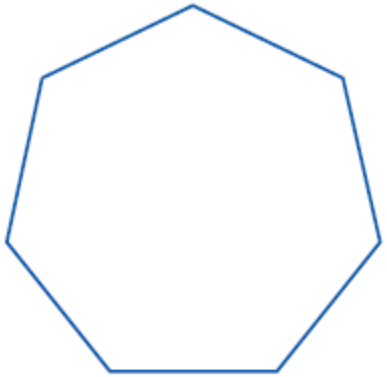
Regular polygons

- A polygon that has all sides of equal length and all angles of equal measure is called a regular polygon.

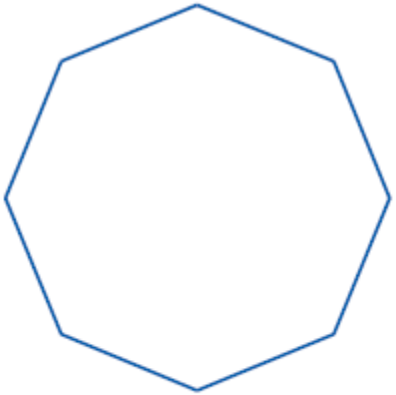


regular triangle | regular quadrangle | regular pentagon | regular hexagon

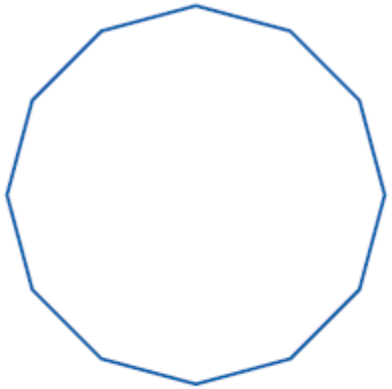
Regular polygons



regular
heptagon

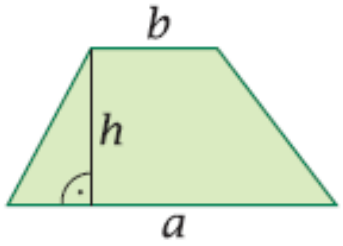


regular
octagon

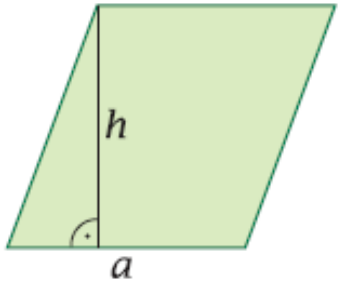


regular
dodecagon

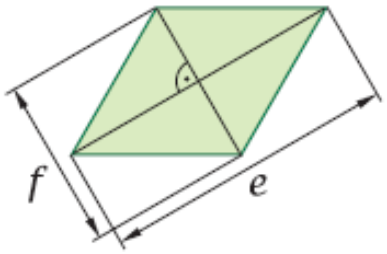
Area of polygons



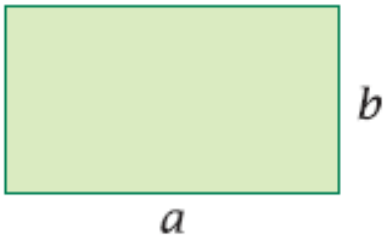
$$A = \frac{(a + b) \times h}{2}$$



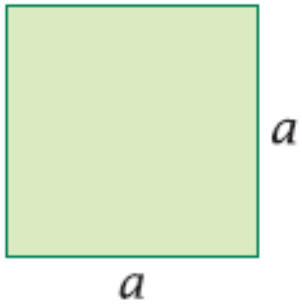
$$A = a \times h$$



$$A = \frac{e \times f}{2}$$



$$A = a \times b$$

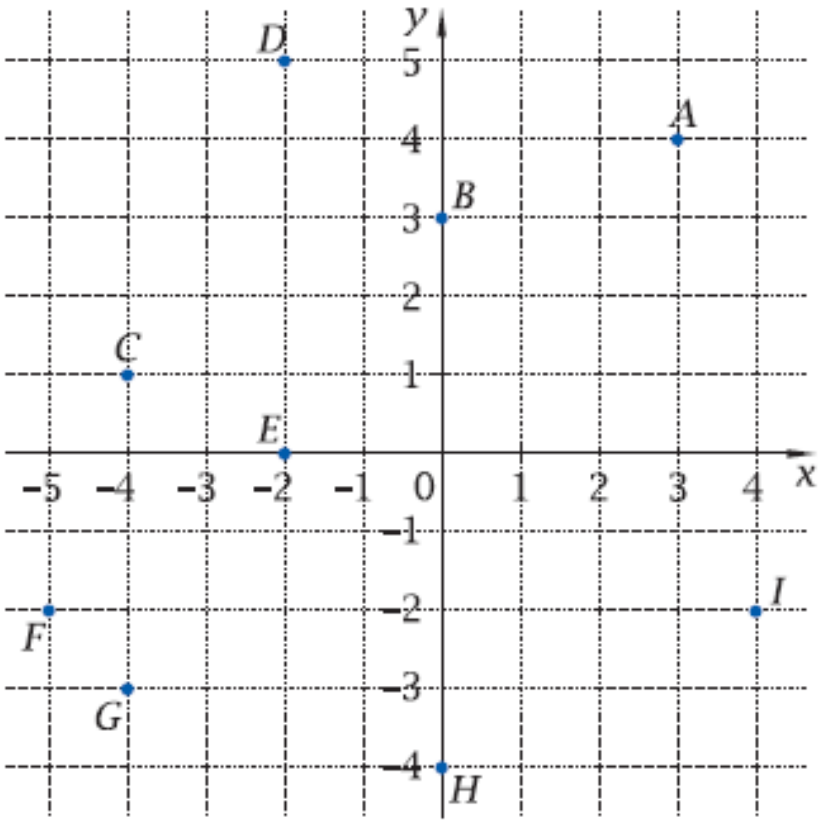


$$A = a^2$$

Coordinate system

- Axes of the coordinate system are perpendicular. The point of intersection of the axes is called the origin of the coordinate system.
- In the coordinate system the position of each point in the plane is determined by two numbers, called coordinates of this point. The first number is the x-coordinate, read on the horizontal axis. The second number is the y-coordinate, read on the vertical axis.

Coordinate system

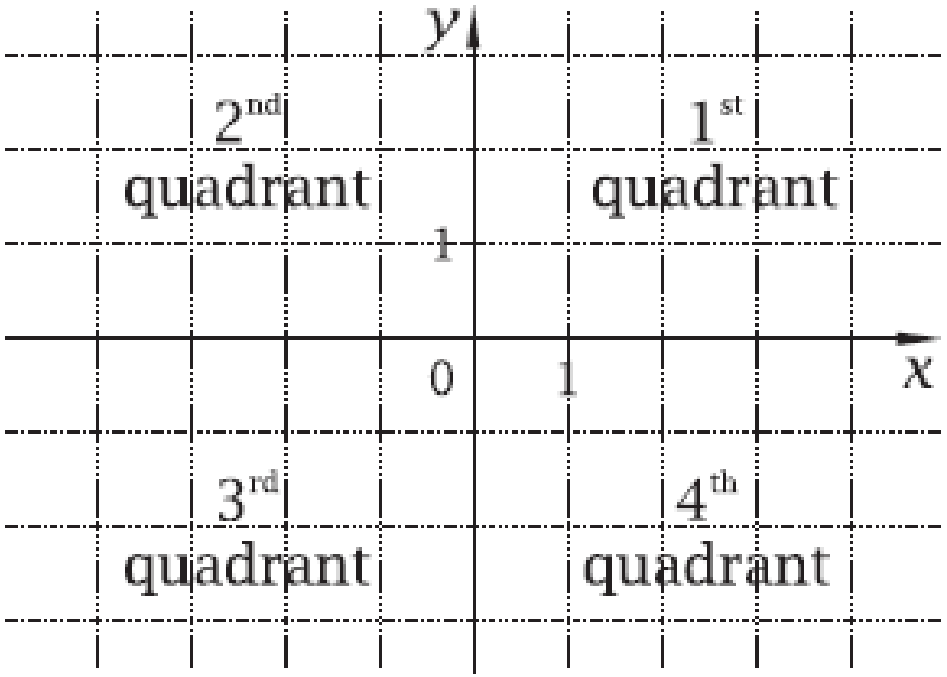


$A = (3, 4)$ $\qquad$ $B = (0, 3)$

x-coordinate of *A* $\qquad$ *y*-coordinate of *A* $\qquad$ *x*-coordinate of *B* $\qquad$ *y*-coordinate of *B*

Coordinate system

- Axes of the coordinate system divide the plane into four parts, called quadrants of the system.



Algebraic expressions

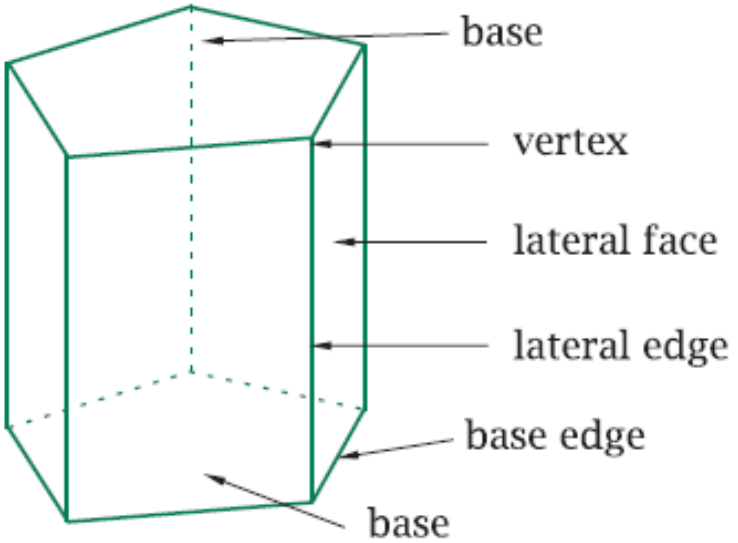
- Expressions in which besides numbers and operation signs occur letters are called algebraic expressions.
- For letters occurring in an algebraic expression we can substitute numbers. We then get an arithmetic expression.
- Such basic expressions, which consist of single numbers, letters or numbers and letters are called monomials.
- The number occurring at the beginning of the ordered monomial is called the coefficient of the monomial.

Equations

- Each number satisfying the equation is called a solution to this equation.
- An equation, which is satisfied by all numbers is called identity equation.
- An equation, which is not satisfied by any number is called inconsistent equation.
- The set of all numbers satisfying a given equation is called the solution set to that equation.
- Two equations having the same set of solutions are called equivalent equations.

PRISMS AND PYRAMIDS

- The following figure shows a right prism. Such a prism has two bases that are parallel congruent polygons, and its lateral faces are rectangles.



right pentagonal prism

PRISMS AND PYRAMIDS

- A right prism whose base is a regular polygon is called a regular prism.

Volume of a prism: $V = A_b \times H$

A_b — area of the base

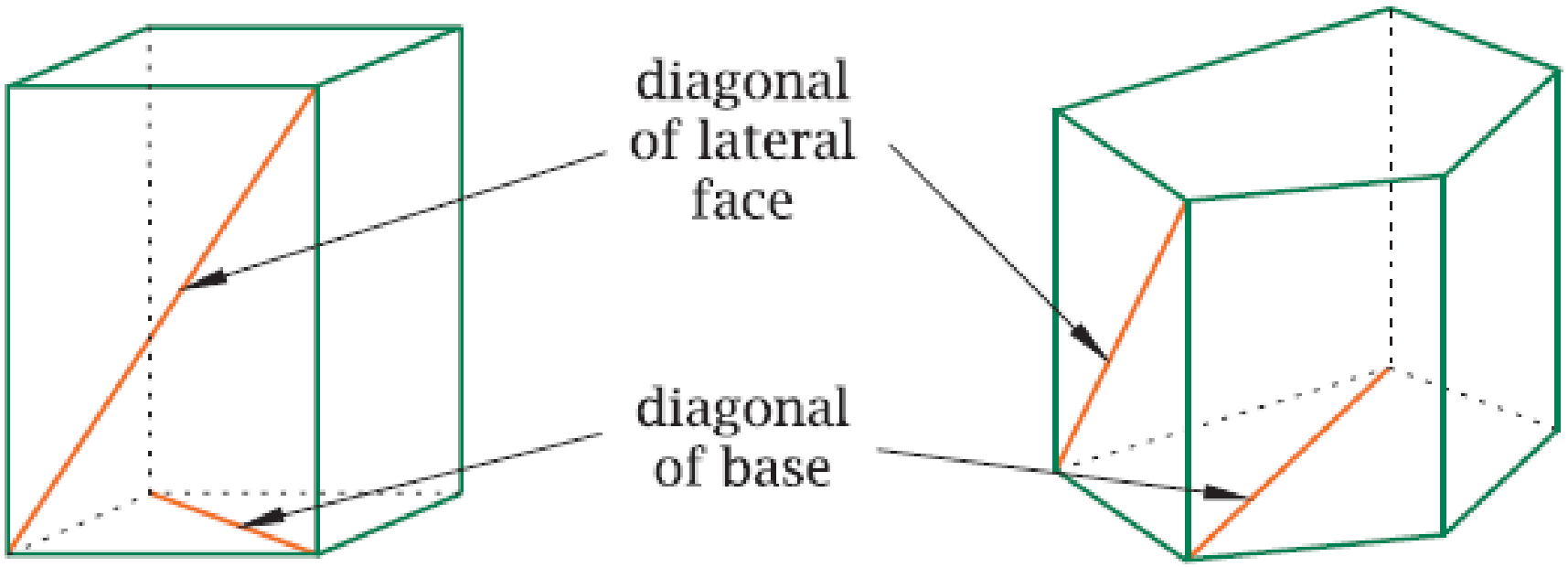
H — height of the prism

Total surface area: $A = 2A_b + A_l$

A_b — area of the base

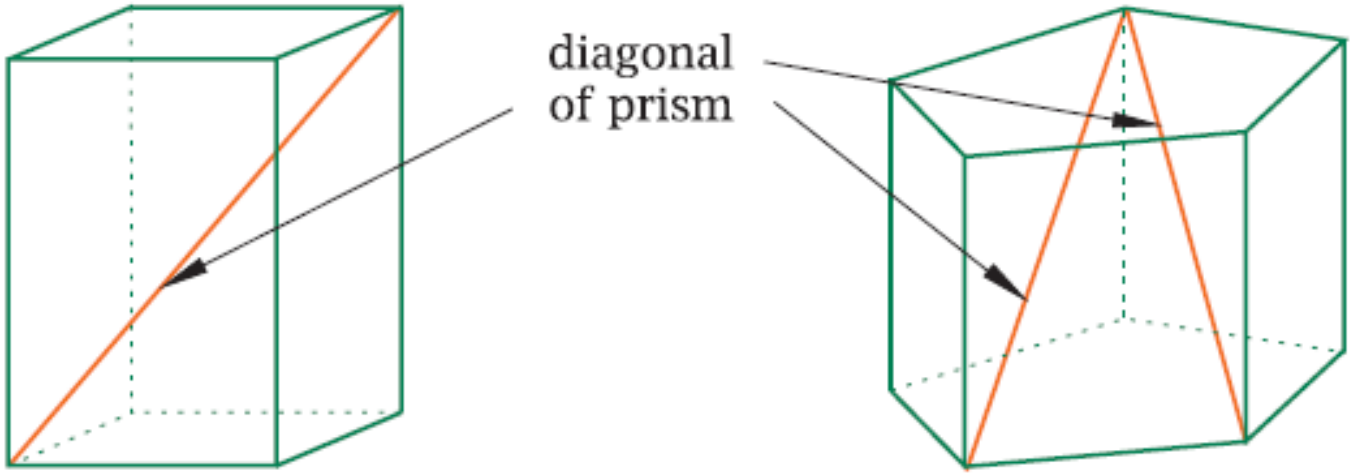
A_l — lateral area (sum of the areas of all lateral faces)

Line segments in prisms



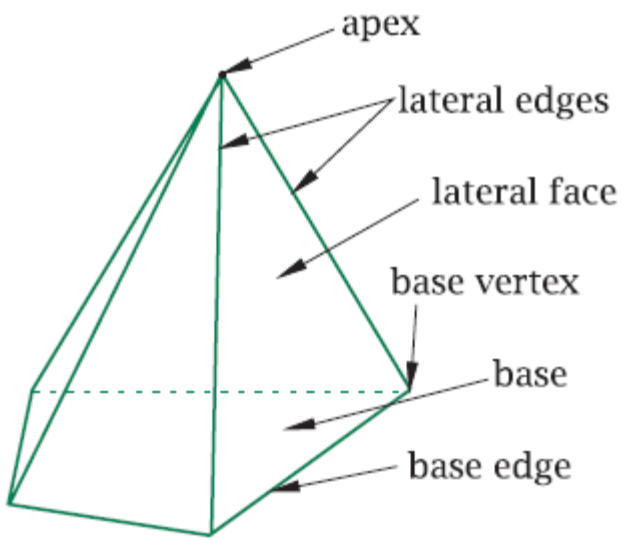
Line segments in prisms

- A line segment that connects two vertices of a prism and is not contained in none of its faces, we will call the diagonal of the prism.



Types of pyramids

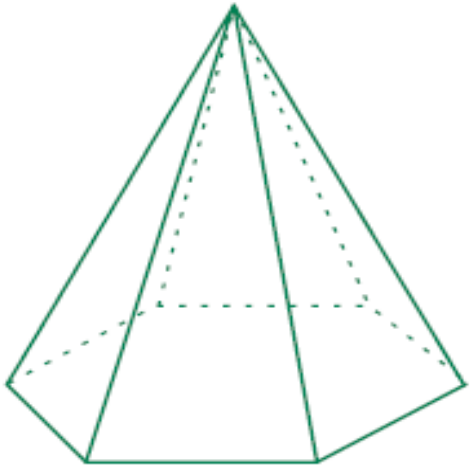
- In each pyramid the base is a polygon, and the lateral faces are triangles.
- The common vertex of lateral faces we call the apex of the pyramid.
- The triangular pyramid is also called a tetrahedron.



quadrangular pyramid

Types of pyramids

- If the base of a pyramid is a regular polygon and lateral edges have equal length, we call it a regular pyramid.
- The pyramid whose all faces are equilateral triangles, we call the regular tetrahedron.



regular hexagonal pyramid

Pyramid

Pyramid surface area: $A_{pyr} = A_b + A_l$

A_{pyr} — total surface area
A_b — the base area
A_l — lateral surface area

The volume of pyramid: $V = \frac{1}{3} A_b \times H$

A_b — the pyramid's base area
H — height of the pyramid